Stabilization of a Premixed Planar Flame by Localized Energy Addition

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The conceptual stabilization against blowoff, of a steady planar flame in a homogeneous gaseous premixture flowing at a speed in excess of the adiabatic flame speed, is examined theoretically. The stabilization is achieved by means of a highly localized, continuous, nonintrusive enthalpy deposition, uniformly in a plane downwind of the flame position. The position of the flame relative to the enthalpy source is found as a function of the strength of the source (or, equivalently, of the magnitude by which the hot-boundary temperature exceeds the adiabatic flame temperature). This theoretically achievable stabilization seems pertinent only for fundamental laboratory investigations, because the source must heat every fluid element of the flowing stream. It is noted that, with less energy input, a continuous point (or line) source of energy could stabilize an axisymmetric (or planar-symmetric), semi-infinite bluff-body-configured flame, because only that portion (of the flowing premixture) that passes near to the source would (and need) be augmented in enthalpy.

Nomenclature

A = energy of source per area per time
a = $a_1/(\kappa/u_u)$
a_1 = position of source
c_p = specific heat capacity of mixture
D_2 = $QY_{Fu}/[c_p(T_f-T_u)]$
D_F = diffusion coefficient for fuel species
D_O = diffusion coefficient for oxidizing species
h_F^o = specific heat of formation of fuel species at standard conditions
h_O^o = specific heat of formation of oxidizing species at standard conditions
h_P^o = standard heat of formation of product species at standard conditions
I = $A/[\dot{m}c_p(T_f-T_u)]$
k = thermal conductivity
M = $(\rho_u\bar{k}_f)(\kappa Y_{Ou})/u_u^2$
m = $m_O\nu_O+m_F\nu_F$
m_F = molecular weight of fuel species
m_O = molecular weight of oxidizing species
m_P = molecular weight of (lumped) product species
$\dot{m}$ = $\rho_u u_u$
$\dot{m}_n$ = $\rho_u(u_u)_n$
Q = $(h_O^o\nu_O m_O+h_F^o\nu_F m_F-h_P^o\nu_P m_P)/m$
T = temperature
u = flow speed
X = Y_O/Y_{Ou}
x = spatial coordinate
Y = Y_F/Y_{Fu}
Y_F = $m\tilde{Y}_F/(m_F\nu_F)$
Y_O = $m\tilde{Y}_O/(m_O\nu_O)$
$\tilde{Y}_F$ = fuel species mass fraction
$\tilde{Y}_O$ = oxidizing species mass fraction
$\tilde{Y}_P$ = (lumped) product species mass fraction
α^{-1} = $(T_f-T_u)/T_u$
β = $\theta/(T_f-T_u)$
δ = $(\kappa/u_u)\delta_1$
δ_1 = Dirac delta function
θ = Arrhenius activation temperature
κ = $k/(\rho_u c_p)$
λ = $\dot{m}_n/\dot{m}$, or $(u_u)_n/u_u$
ν_F = stoichiometric coefficient for fuel species
ν_O = stoichiometric coefficient for oxidizing species
ν_P = stoichiometric coefficient for (lumped) product species
ρ = density
$\rho_u\bar{k}_f$ = effective frequency factor
σ_F = Lewis-Semenov number for fuel species, $\rho D_F/(k/c_p)$
σ_O = Lewis-Semenov number for oxidizing species, $\rho D_O/(k/c_p)$
τ = $(T-T_u)/(T_f-T_u)$
ϕ = Y_{Fu}/Y_{Ou}
χ = $x/(\kappa/u_u)$

Subscripts

b = evaluated at the hot boundary ($x\to\infty$)
F = fuel species
f = evaluated at the hot boundary ($x\to\infty$) for source-free conditions
O = oxidizing species
P = (lumped) product species
u = evaluated at the cold boundary ($x\to-\infty$)
$(\ )_n$ = pertinent to "nonadiabatic" conditions (i.e., with a heat source or sink)

I. Introduction

THIS paper is the first in a series that theoretically considers the initiation and early evolution of burning as a consequence of *nonintrusive* enthalpy deposition in a homogeneous combustible premixture flowing supercritically (i.e., at a speed in excess of the adiabatic flame speed). The series seeks to explore the stabilization of flames against blowoff by use of existing optical sources, and of other sources that plausibly may exist in the near future. Thus, several well-known aerothermochemical phenomena are to be reconsidered in the light of recent technological advances.

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While attention in later papers is to be centered on *intermittent* enthalpy deposition (e.g., from rapidly repeatedly pulsed lasers), in this first paper a *continuous* planar source, for stabilizing a steady one-dimensional flame in space, is considered. For such a conceptual burner, the burned-gas (hot-boundary) temperature exceeds the adiabatic flame temperature. In fact, the burner is the counterpart of the conventional (so-called) flat-flame burner, which extracts heat *intrusively* from a subcritically flowing premixture to stabilize a one-dimensional flame against flashback, so the burned-gas temperature is below the adiabatic flame temperature.[1] Indeed, the conceptual burner to be studied here is conceived as a laboratory device, since preheating every fluid element (as a planar source must) is very enthalpy-demanding. The postulated planar optical source may someday exist as an alternative to the adding of radiatively absorbing-emitting particles to the gaseous premixture,[2] having the reaction occur in highly thermally conducting metallic tubes,[3] or adopting other intrusive schemes[4] for stabilizing supercritical planar flames. However, the major motivation for this study is to develop results for comparison with results from later studies.

II. Formulation

Under the model of a direct one-step irreversible reaction

$$\nu_O O + \nu_F F \rightarrow \nu_P P \tag{1}$$

a steady one-dimensional quasi-isobaric solution is sought to the conservation laws in a slightly generalized Shvab-Zeldovich approximation.[5] For a flame-fixed coordinate, the boundary eigenvalue problem may be written (for a nonintrusive heat source of given magnitude I at $\chi = a$, where a is to be found) dimensionlessly as

$$\lambda Y_\chi - \sigma_F Y_{\chi\chi} = -MYX \exp[-\beta/(\alpha^{-1}+\tau)] \tag{2a}$$

$$\lambda(X/\phi)_\chi - \sigma_O (X/\phi)_{\chi\chi} = -MYX \exp[-\beta/(\alpha^{-1}+\tau)] \tag{2b}$$

$$\lambda\tau_\chi - \tau_{\chi\chi} = D_2 MYX \exp[-\beta/(\alpha^{-1}+\tau)] + I\delta(\chi - a) \tag{2c}$$

where (X_b, τ_b are constants bo be found; $\chi \rightarrow -\infty$ at the cold boundary, $\chi \rightarrow \infty$ at the hot boundary)

$$\chi \rightarrow -\infty: \quad Y \rightarrow 1, \quad X \rightarrow 1, \quad \tau \rightarrow 0 \tag{3a}$$

$$\chi \rightarrow \infty: \quad Y \rightarrow 0, \quad X \rightarrow X_b, \quad \tau \rightarrow \tau_b \tag{3b}$$

Equations (2a-2c) express the conservation of fuel, oxidizer, and energy, respectively, for a bimolecular second-order Arrhenius-type reaction rate (β, α^{-1} given); generalization of the formulation (or solution below) to other orders of reaction is straightforward. By Eq. (3b), attention is limited to fuel-lean ($X_b > 0$) or stoichiometric ($X_b = 0$) conditions, though (soot-free) fuel-rich conditions are included by interchanging the definitions of X, Y.

The given formulation holds for given constant (though general) values for the Lewis-Semenov numbers σ_O, σ_F (here, the ratio of the diffusion coefficient of a particular species to the thermal diffusivity κ, with the thermal conductivity and specific heat capacity of the mixture being held constant); the constancy of σ_O, σ_F implies that each diffusion coefficient (D_O, D_F) varies linearly with temperature. Also, $\rho T \sim$ const.

By forming a linear combination of Eqs. (2a) and (2c), such that the transcendentally nonlinear reaction-rate term is eliminated, then integrating over the full infinite domain in χ and using Eqs. (3a) and (3b) gives

$$\tau_b = D_2 + I/\lambda \tag{4}$$

For $I = 0$, $\lambda = 1$, and $\tau_b = 1$, by choice of nondimensionalization, so in general $D_2 = 1$. Hence, the well-known result

$$T_f = T_u + (Q/c_p) Y_{Fu} \Rightarrow \tau_b = 1 + I/\lambda \tag{5}$$

Similarly, Eqs. (2a) and (2b) yield ($1 \geq \phi$, for cases of interest)

$$X_b = 1 - \phi \Rightarrow Y_{Ob} = Y_{Ou} - Y_{Fu} \tag{6}$$

For ϕ "small enough", $X_b \rightarrow 1$, so plausibly $X \rightarrow 1$ for all χ. First-order kinetics then holds: complete conversion of the leaner reactant hardly depletes the field of the richer reactant. In general, for $\sigma_O = \sigma_F$,

$$X = \phi Y + X_b + \phi Y = (1 - \phi) \tag{7}$$

All parameters typically are $O(1)$, except 1) typically $\beta \gg 1$, and 2) the value of M is to be inferred. The largeness of β implies a structure in which, to lowest order of approximation, a relatively thin reactive-diffusive layer lies amid broader expanses in which convective-diffusive balances hold.[6] Because the problem posed by Eqs. (2a-2c) and (3a) and (3b) is translationally invariant, the location of the thin diffusive-reactive layer may be (and, for convenience, is) assigned a value $\chi = 0$. With this assignment, the standoff distance of the source a is meaningfully defined.

The parameter M is identified by reconstructing the classical case $\lambda = 1$, $I = 0$, for which M is an eigenvalue (its factor u_u is to be identified). Henceforth, M is a known parameter; $\lambda(>1$, for cases of interest here) is assigned; and $I(>0$, for cases of interest here) is also assigned. If a solution to a postulated set of parametric assignments can be found (e.g., if a value can be assigned to the site a), then the case is consistent with the conservation laws in the Shvab-Zeldovich approximation.

Incidentally, the adopted nondimensionalization is elegant and convenient for investigating the consequences (of assigning various values to I, λ) on (say) source position and reactive zone temperature, for a given premixture. However, the nondimensionalization may be inconvenient for contrasting the response of alternative premixtures for fixed values of I, λ. Thus, it well may be modified in sequential papers in this series.

Finally, the formulation [Eqs. (2a-c) and (3a) and (3b)] encounters the well-known cold-boundary difficulty, as $\chi \rightarrow -\infty$ unless $(1/\alpha) \rightarrow 0$. However, for approximate solution with $\beta \gg 1$, this detail is no impediment.

III. Eigenvalue

We identify the eigenvalue M by treating the case $I = 0$, $\lambda = 1$. For $\beta \gg 1$ and for $\chi < 0$,

$$\tau(\chi) = \exp(\chi), \qquad Y(\chi) = 1 - \exp(\chi/\sigma_F),$$
$$X(\chi) = 1 - \phi \exp(\chi/\sigma_O) \tag{8}$$

for $\chi > 0$,

$$\tau(\chi) = 1, \qquad Y(\chi) = 0, \qquad X(\chi) = 1 - \phi \tag{9}$$

Near $\chi = 0$, i.e., for the thin reactive-diffusive layer,

$$(\phi\tau + \sigma_O X)_{\chi\chi} \doteq 0 \Rightarrow X \doteq (1-\phi) + (1-\tau)/(\sigma_O/\phi) \tag{10a}$$

$$(\tau + \sigma_F Y)_{\chi\chi} \doteq 0 \Rightarrow Y \doteq (1-\tau)/\sigma_F \tag{10b}$$

$$-\tau_{\chi\chi}\tau_\chi \doteq M\left[\frac{1-\tau}{\sigma_F}\right]\left[(1-\phi) + \frac{1-\tau}{\sigma_O/\phi}\right]\left[\exp\left(-\frac{\beta}{(1/\alpha)+\tau}\right)\right]\tau_\chi \tag{10c}$$

By integrating from $\chi = 0-$, where $\tau_\chi = 1$, to $\chi = 0+$, where $\tau_\chi = 0$,

$$-(\tau_\chi^2/2)\Big|_{\chi=0-}^{\chi=0+} = \frac{1}{2}$$

$$= M\left\{\frac{1-\phi}{\sigma_F} I_1(\alpha,\beta) + \frac{\phi}{\sigma_O \sigma_F} I_2(\alpha,\beta)\right\} \tag{11}$$

where, for $n = 1,2$,

$$I_n(\alpha,\beta) = \int_{1-}^{1} (1-\tau)^n \exp\left(-\frac{\beta}{(1/\alpha)+\tau}\right) d\tau$$

$$\doteq n \frac{[(1/\alpha)+1)]^{2(n+1)}}{\beta^{n+1}} \exp\left[-\frac{\beta}{(1/\alpha)+1}\right] \tag{12}$$

To obtain the last relation in convenient, closed-form (but rough) approximation, we replaced the lower limit $\tau = 1-$ with $\tau = 0$, since the integrand is sharply peaked near $\tau = 1$ for $\beta \gg 1$, and we expanded the argument of the exponential about $\tau = 1$. The first term on the right-hand side of Eq. (11) is dominant if $(1-\phi) \gg O(1/\beta)$. This includes the first-order case for which $\chi \doteq 1$ everywhere and $(1-\phi) \doteq 1$. The second term is dominant in the near-stoichiometric case $(1-\phi) \ll O(1/\beta)$.

IV. Simple Nonadiabatic Case

If $\lambda, I > 0$ (and known), then, for $X \to 1$ (i.e., for ϕ small), if $\sigma_F = 1$ and if D is a constant of integration, from Eqs. (2a), (2c), (3a), (3b), and (5),

$$\tau + Y = 1 + I/\lambda, \quad \chi > a \tag{13a}$$

$$\tau + Y = 1 + (D-1)\exp(\lambda\chi), \quad \chi < a \tag{13b}$$

Also,

$$-\infty \le \chi \le 0: \quad Y = 1 - \exp(\lambda\chi)$$

$$\Rightarrow \tau = D \exp(\lambda\chi) \tag{14a}$$

$$0 \le \chi \le a: \quad Y = 0$$

$$\Rightarrow \tau = 1 + (D-1)\exp(\lambda\chi) \tag{14b}$$

$$a \le \chi \le \infty: \quad Y = 0$$

$$\Rightarrow \tau = 1 + I/\lambda \tag{14c}$$

$$\chi \doteq 0: \quad \tau_{\chi\chi}\tau_\chi \doteq -MY\left\{\exp\left[-\frac{\beta}{(1/\alpha)+\tau}\right]\right\}\tau_\chi, \qquad Y \doteq D - \tau \tag{14d}$$

Since $\tau_\chi(0-) = D\lambda$, $\tau_\chi(0+) = \lambda(D-1)$, then, by integration of Eq. (14d) over all χ

$$\lambda^2(2D-1) = 2M \frac{\exp\left\{-\beta/[(1/\alpha)+D]\right\}}{\left\{\beta/[(1/\alpha)+D]^2\right\}^2}$$

$$= \left(\frac{\alpha^{-1}+D}{\alpha^{-1}+1}\right)^4 \exp\left[\frac{\beta(D-1)}{(\alpha^{-1}+1)(\alpha^{-1}+D)}\right] \tag{15}$$

by use of Eqs. (11) and (12). Equation (15) gives the flame temperature $D(>1)$ as a function of (just) three parameters α^{-1}, β, and $\lambda(>1)$. By enforcing continuity at $\chi = a$,

$$a = \frac{1}{\lambda} \ln\left[\frac{I}{\lambda(D-1)}\right] \tag{16}$$

Note that $a < 0$ implies $D > [1 + (I/\lambda)]$, but there is no physical way by which the flame temperature can exceed the hot-boundary temperature (2.7b). Hence, the position of the source is coincident with, or downwind of, the flame:

$$a \ge 0 \quad \text{as} \quad [1 + (i/\lambda)] \ge D.$$

V. Energy Extraction

Whereas energy addition by a source downwind of the reactive layer is the subject of the remainder of this paper, *in this section only,* the more familiar stabilization of a subcritically flowing premixture ($\lambda < 1$) by heat extraction ($I \to -I$, $I > 0$), by introduction of a sink located upwind of the reactive layer ($a \to -a$, $a > 0$), is considered for contrast. Again, for simplicity, $\alpha_F = 1$, and $X \to 1$ (i.e., ϕ is small). But now $\tau \to [1-(I/\lambda)]$ at the hot boundary, where $1 \gg (I/\lambda)$ for cases of interest.

One finds, under the same approximations of Section 4,

$$-\infty \le \chi \le -a: \quad \tau = D\exp(\lambda\chi), \quad Y = 1 - \exp(\lambda\chi); \tag{17}$$

$$-a \le \chi \le 0: \quad \tau = \exp(\lambda\chi) - (I/\lambda), \quad Y = 1 - \exp(\lambda\chi); \tag{18}$$

$$0 \le \chi \le \infty: \quad \tau = 1 - I/\lambda, \quad Y = 0; \tag{19}$$

$$\lambda^2 = \left[1 - \frac{(I/\lambda)}{\alpha^{-1}+1}\right] \exp\left[\frac{\beta(I/\lambda)}{(\alpha^{-1}+1)(\alpha^{-1}+1-I/\lambda)}\right]; \tag{20}$$

$$a = \frac{1}{\lambda} \ln\left(\frac{1-D}{I/\lambda}\right). \tag{21}$$

For $a > 0$, i.e., for the sink to lie upwind of the reactive layer, the temperature at the sink D must be less than the hot boundary temperature, $[1-(I/\lambda)]$. But Eq. (20) requires a relation between the sink strength I and the flow speed λ; further, while Eq. (21) relates D and a, neither is constrained to a value. Thus, anomalously, the magnitude of the sink I is constrained, but the site of the sink a is widely assignable. No physical way to create the postulated heat sink is known to the authors, and the solution reflects the imaginary nature of the process.

However, a solution is readily found for a simplistic model of a conductive-type (inherently intrusive) heat sink, such as a porous disk with embedded water-circulating coils. The last term of Eq. (2c) is rewritten as $-K(\tau - \tau_h)\delta(\chi + a)$, where $a(>0)$, the site of the sink, and $\tau(-a)$, the temperature of the gas at the sink, are to be found. The heat-transfer coefficient $K_1(>0)$, in heat per area per time per degree Kelvin, and the temperature of the sink τ_h (with $1 > \tau_h > 0$), are given; then,

$$K \equiv \frac{K_1}{\rho_u u_u c_p}, \qquad \tau_h \equiv \frac{T_h - T_u}{T_f - T_u} \tag{22}$$

presumably, $\tau(-a) > \tau_h$. Again for $\sigma_F = 1$, $X \to 1$,

$$0 \le \chi \le -a: \quad \tau(-a) = \exp[\lambda(\chi + a)]$$
$$Y = 1 - \exp(\lambda\chi) \tag{23a}$$

$$-a \le \chi \le 0: \quad \tau = \exp(\lambda\chi) - (K/\lambda)[\tau(-a) - \tau_h]$$
$$Y = 1 - \exp(\lambda\chi) \tag{23b}$$

$$0 \le \chi \le \infty: \quad \tau = 1 - (K/\lambda)[\tau(-a) - \tau_h] \equiv b, \qquad Y = 0 \tag{23c}$$

$$\tau(-a) = \frac{\exp(-\lambda a) + (K/\lambda)\tau_h}{1 + (K/\lambda)} \tag{23d}$$

$$\lambda^2 = \left(\frac{\alpha^{-1}+b}{\alpha^{-1}+1}\right)^4 \exp\left[-\frac{\beta(1-b)}{(\alpha^{-1}+1)(\alpha^{-1}+b)}\right] \tag{23e}$$

Equation (23e) yields $\tau(-a)$; Eq. (23d) then gives the sink site a. For $a > 0$, $\tau(0) > \tau(-a)$; for $b < 1$, $\lambda < 1$ and $\lambda a < \ln(1/\tau_h)$.

VI. Energy Addition: More General Conditions

The relaxing of restrictions of Sec. IV, so that σ_O and σ_F may assume nonunity values and ϕ need not be small, is now shown to alter the profiles of the dependent variables, but in general not to alter significantly many key properties (source position, reactive-zone temperature). The approximate solution to Eqs. (2a-2c), (3a) and (3b), and (4-6) becomes, under Eqs. (11) and (12),

$$-\infty \le \chi \le 0:\ \tau = D\exp(\lambda\chi), \qquad Y = 1-\exp(\lambda\chi/\sigma_F)$$
$$X = 1-\phi\exp(\lambda\chi/\sigma_O) \tag{24a}$$

$$0 \le \chi \le a:\ \tau = \frac{D-1-(I/\lambda)}{1-\exp(\lambda a)}\exp(\lambda\chi) + \frac{1+(I/\lambda)-D\exp(\lambda a)}{1-\exp(\lambda a)}, \qquad Y=0,\ X=1-\phi \tag{24b}$$

$$a \le \chi \le \infty:\ \tau = 1+(I/\lambda), \qquad Y=0,\ X=1-\phi \tag{24c}$$

$$\chi \doteq 0:\ Y \doteq \frac{D-\tau}{\sigma_F}, \qquad X \doteq \frac{D-\tau+(\sigma_O/\phi)(1-\phi)}{\sigma_O/\phi} \tag{24d}$$

$$-(\tau_\chi)^2\Big|_{\chi=0-}^{\chi=0+} \doteq 2M\int_0^D YX\exp\left[-\frac{\beta}{(1/\alpha)+\tau}\right]\mathrm{d}\tau \tag{24e}$$

$$\lambda^2(2D-1) \doteq \left(\frac{\alpha^{-1}+D}{\alpha^{-1}+1}\right)^4\cdot\left[\frac{2(\alpha^{-1}+D)^2+\sigma_O(1-\phi)\beta/\phi}{2(\alpha^{-1}+1)^2+\sigma_O(1-\phi)\beta/\phi}\right] \times\exp\left[\frac{\beta(D-1)}{(\alpha^{-1}+1)(\alpha^{-1}+D)}\right] \tag{24f}$$

$$a = \frac{1}{\lambda}\ln\left[\frac{(I/\lambda)}{D-1}\right] \tag{24g}$$

Only if $[\sigma_O(1-\phi)\beta/\phi] = O(1)$ does this solution differ much from that of Sec. IV; in general, a distinction arises only if $(1-\phi) = O(\beta^{-1})$. As in Sec. IV, one expects $[1+(I/\lambda)] > D > 1$, so $a > 0$.

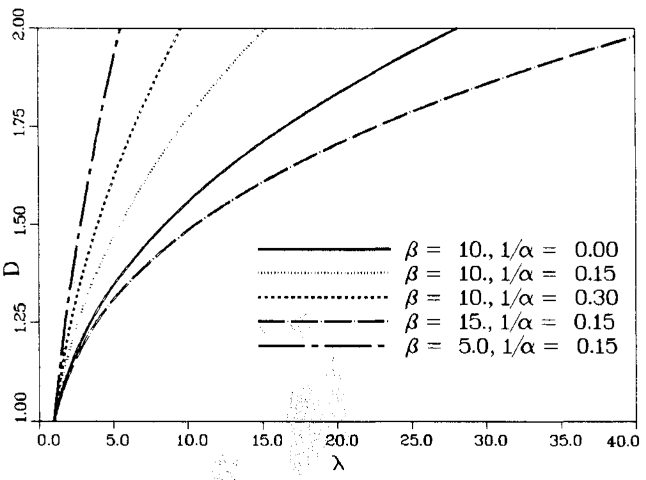

Fig. 1 Source-enhanced flame temperature D, normalized so that the adiabatic flame temperature is unity, vs the cold-flow speed λ, normalized so that the adiabatic flame speed is unity, for several values of the normalized activation temperature β and the burned-gas-temperature-increment/cold-gas-temperature ratio α^{-1}, from Eq. (15).

VII. Flame Stabilization in a Supercritical Flow by Heat Conduction from a Source

For curiosity, Eq. (2c) is rewritten [K, τ_h defined as in Eq. (22), though now $\tau_h > 1$]

$$\lambda\tau_\chi - \tau_{\chi\chi} = MYX\exp[-\beta/(\alpha^{-1}+\tau)] + K[\tau_h-\tau]\delta(\chi-a) \tag{25}$$

This equation is solved with $K>0$, $a>0$, $\tau_h > \tau(a)$, together with Eqs. (2b-2c) and (3a) and (3b) to examine the use of heat addition by conduction for stabilization against blowoff of a flame in a supercritical flow. Whereas Eq. (6) still holds, Eq. (5) is modified: integration of Eq. (25) over all χ yields

$$\tau_b = 1+\frac{K}{\lambda}\Big[\tau-\tau(a)\Big] \Rightarrow \tau_b + \frac{1+(K/\lambda)\tau_h}{1+(K/\lambda)} \tag{26}$$

since $\tau(a) = \tau_b$. Since heat addition is anticipated to be required for stabilization, we required $\tau_h > 1$, as noted. However, requiring that the heat reservoir be maintained at a temperature in excess of the adiabatic flame temperature reduces the concept to one of academic interest.

The solution is

$$-\infty \le \chi \le 0:\ \tau = D\exp(\lambda\chi), \qquad Y = 1-\exp(\lambda\chi/\sigma_F)$$
$$X = 1-\phi\exp(\lambda\chi/\phi) \tag{27a}$$

$$0 \le \chi \le a:\ \tau = \frac{\tau_b - D}{\exp(\lambda a)-1}\exp(\lambda\chi) + \frac{D\exp(\lambda a)-\tau_b}{\exp(\lambda a)-1}$$
$$Y=0, \qquad X = 1-\phi \tag{27b}$$

$$a \le \chi \le \infty:\ \tau = \tau_b, \qquad Y=0, \qquad X=1-\phi \tag{27c}$$

Near $\chi = 0$, relations analogous to Eqs. (24d) and (24e) yield

$$\lambda^2(2D-1) = \left(\frac{\alpha^{-1}+D}{\alpha^{-1}+1}\right)^4\cdot\left[\frac{(\alpha^{-1}+D)^2+\sigma_O(1-\phi)\beta/\phi}{(\alpha^{-1}+1)^2+\sigma_O(1-\phi)\beta/\phi}\right] \times\exp\left[\frac{\beta(D-1)}{(\alpha^{-1}+D)(\alpha^{-1}+1)}\right] \tag{28}$$

Requiring appropriate continuity at $\chi = a$, upon use of Eq. (26) gives

$$a = \frac{1}{\lambda}\ln\left\{\frac{(K/\lambda)(\tau_h-1)}{[1+(K/\lambda)](D-1)}\right\} \tag{29}$$

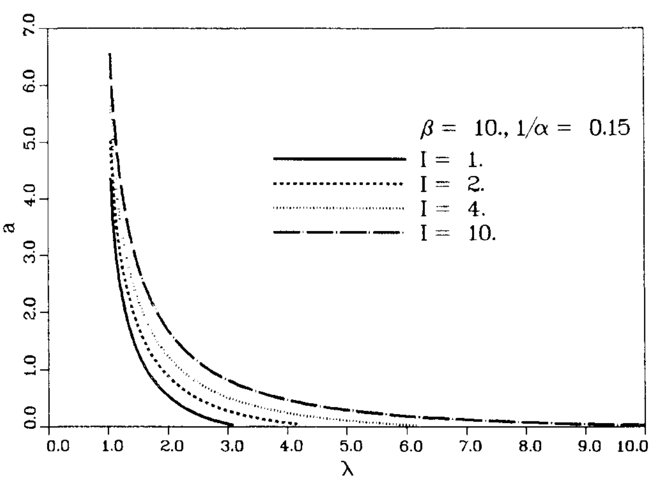

Fig. 2 Distance a from flame to source, nondimensionalized against the heat-diffusion scale, vs the normalized cold-flow speed λ, for several values of the normalized source strength I, from Eq. (16).

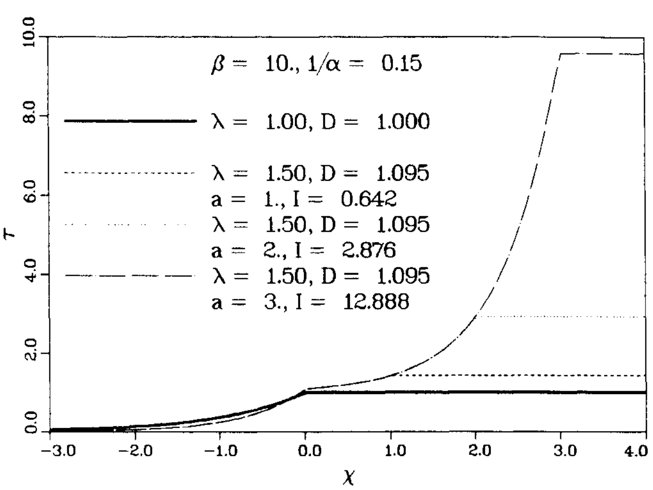

Fig. 3 Dimensionless temperature τ vs dimensionless spatial coordinate χ with the flame at $\chi=0$, for several values of the normalized source strength I, from Eqs. (14a-14c). The dark curve is the classical adiabatic profile.

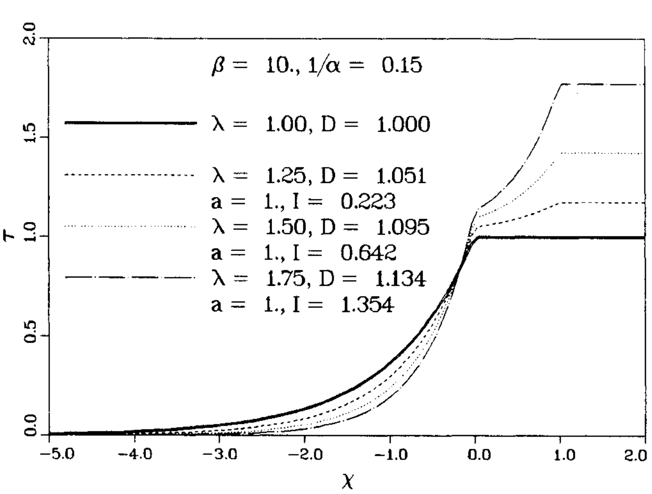

Fig. 4 Further spatial profiles of the normalized temperature τ, for various values of the dimensionless source/flame displacement a and the dimensionless source strength I.

One anticipates $\tau_h > \tau_b > 1$, so $a > 0$.

Although K has been taken as constant here, accounting for a dependence of K on λ, i.e., on the flow speed, would be part of an improved model.

VIII. Numerical Examples

For plausible numerical values, i.e., $\beta = O(10)$, $\alpha^{-1} = O(0.1)$, $\lambda = O(3)$, values of the normalized flame temperature D, obtained from Eq. (15) for the simple heat-addition case, are presented as a function of the normalized cold-boundary speed λ in Fig. 1. For fixed values of the dimensionless activation temperature β, and of the ratio of the cold-boundary temperature to the adiabatic combustion-engendered temperature rise α, D increases monotonically with λ. The position of the associated source site a, relative to the flame (invariably assigned a nominal position at the origin), is found (Fig. 2) to decrease monotonically as the cold-boundary speed increases, according to Eq. (16).

The standoff distance of the flame from the source increases as the nondimensionalized strength of the source I increases. The associated thermal profiles as a function of position χ, as obtained from Eqs. (14a-14c) are presented in Figs. 3 and 4. The dark curve, added as a reference, is for the source-free adiabatic case, in which $D=1$, $\lambda=1$. Fixed standoff distance between source and flame implies a stronger source for a faster flow, and generally a higher hot-boundary temperature. Further, a larger standoff distance for a fixed flame temperature also implies a stronger source.

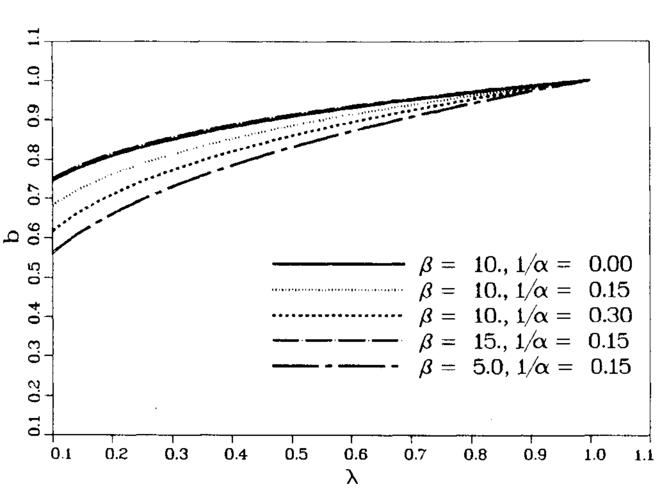

Fig. 5 Sink-decremented flame temperature b, normalized so that the adiabatic flame temperature is unity, vs the normalized cold-flow speed λ, for several values of the dimensionless activation temperature β and of the dimensionless parameter α, related to the hot-boundary/cold-boundary temperature ratio, from Eq. (23e).

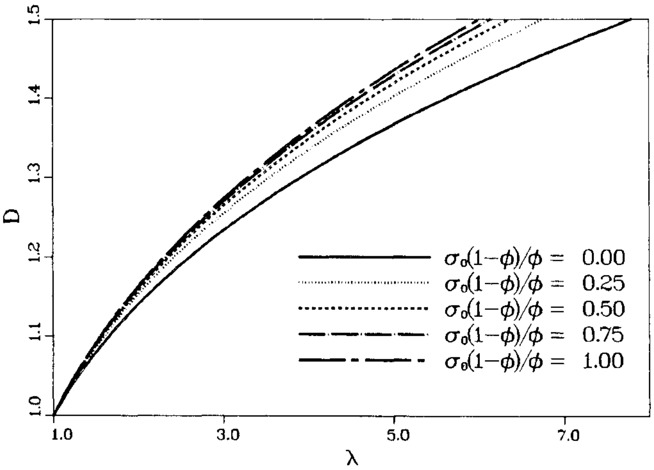

Fig. 6 Source-enhanced flame temperature D, normalized so that the adiabatic flame temperature is unity, vs the normalized cold-flow speed λ, for several values of the parameter $\sigma_O(1-\phi)\phi$, where σ_O is the oxygen-diffusivity/thermal-diffusivity ratio and ϕ is the (fuel-to-oxygen) stoichiometric ratio, from Eq. (24f).

For typical numerical values, dimensionally, standoff distances are typically a few tenths of a millimeter; the source strengths are in the range of 100 W/cm^2; and the hot-boundary temperatures are characterized by values of 2800 K or so. Clearly, for values of the dimensionless flame temperature D approaching 1.5 dissociative effects (reverse reactions) not included in the formulation begin to enter, and eventually so do radiative-transfer effects.

For comparison with the just discussed heat source stabilization of a flame in a supercritical flow, the decrease of the hot-boundary temperature b with decreasing (subcritical) cold-boundary flow speed, as given by Eq. (23e) for the case of a heat sink, is quantified in Fig. 5. The flame standoff distance from the heat sink λa monotonically decreases as λ decreases, all other parameters being held fixed, according to Eq. (23d). In presenting the product λa, the standoff distance is being normalized against a length based on the pertinent subcritical flow speed (the quantity a itself is normalized against a length based on the adiabatic flame speed).

The modification of the flame temperature D owing to stoichiometric and Lewis-number effects, as obtained from Eq. (24f), is presented in Fig. 6. That the modification is not large is evidenced by the fact that, according to Eq. (24g), the standoff distance of the source from the flame is little altered over the range of the parameter $\sigma_O(1-\phi)/\phi$ examined in Fig. 6.

IX. Concluding Remarks

A relatively simplistic, one-step model of the chemical conversion of fuel and oxidant to product gas has been adopted in this study. Whatever the merits of chemically sophisticated models for providing a microscropic description of the flame *structure*, the merits of the model adopted here, with suitably chosen parametric values in the specification of the reaction rate, are equally appropriate for the delineation of the macroscopic features of the overall configuration (with *no* attention to the structure of the flame). In fact, since the flame structure is not at issue in this study, the use of a chemically sophisticated model would have been both unnecessarily cumbersome and digressive.

The nonintrusive stabilization of a planar flame in a supercritically flowing premixture by localized energy deposition seems achievable in the laboratory in the future. However, the nonintrusive stabilization of an axisymmetric, bluff-body-configured flame in a supercritically flowing premixture by means of energy deposition along a "line" or at a "point" seems likely to be demonstrated even sooner. Clearly such a stabilization by a continuous source is less energy-demanding because only that portion of the premixture flowing past the source near the source need be heated, rather than all the premixture (as is required in the planar case presented here). The flame-stabilizing use of continuous (and intermittent) line and point sources of deposited energy is to be the subject of subsequent studies.

With respect to practical considerations as a guide to future related studies, the following observations may be noteworthy. Suppose one employs intermittent (say, periodic) nonintrusive deposition of efficiently absorbed ignition-inducing energy into a very localized volume of a supercritically flowing combustible uniform premixture. If a spherical flame propagation is initiated, in general, less energy input is required to effect burning of the premixture than is required for the stabilization of a flame by a continuous source. However, after a brief time following ignition, the lateral propagation of flame is at the local flame speed, whereas the downwind convection speed may far exceed the flame speed. For purposes of spanning a flowing premixed stream with flame within a limited distance downwind of a given plane, many energy sources would have to be laterally distributed in that plane (or in the vicinity of that plane). But if the deposited energy were sufficient not just to initiate an outwardly propagating flame in the premixture, but even to initiate an outwardly propagating detonation, then a much reduced number of laterally distributed sources would be required to convert the flowing stream of premixed reactants to product gas within a relatively short distance downwind of the sources. In fact, one would want to be certain that the oncoming premixture (taken to be capable of sustaining a detonation) were flowing at sufficiently supersonic speed that the detonations did not propagate upwind. This precaution may require in the future that the energy deposition not be excessive.

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